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FREE CONVECTION FROM A DOWNWARD FACING INCLINED FLAT PLATE

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Abstract—Laminar and turbulent free convection in water from a downward facing inclined flat plate with uniform-heat-flux has been investigated. A system of baffles and a cooler is employed in order to eliminate the constraints of the finite volume of the water tank. Heat-transfer correlations are presented for the laminar and turbulent flow together with transition criteria. Mean boundary-layer temperature profiles and, in the case of turbulent flows, temperature fluctuations are also presented.

NOMENCLATURE

g, gravitational acceleration [m/s^2];
Gr_x^*, local modified Grashof number, $g\beta\dot{q}''x^4/k\nu^2$;
h, local heat-transfer coefficient [W/m^2 K];
k, thermal conductivity [W/mK];
Nu_x, local Nusselt number, hx/k;
Pr, Prandtl number;
$\dot{q}''$, local heat flux density [W/m^2];
Ra_x^*, local modified Rayleigh number, Gr_x^*Pr;
T, temperature [K or °C];
T', RMS fluctuating component of temperature [K];
x, y, coordinates parallel and perpendicular to plate, respectively [m].

Greek symbols

β, thermal expansion coefficient [K^{-1}];
δ_h, $\sim \dfrac{1}{T_w - T_\infty}\displaystyle\int_0^\infty (T - T_\infty)\,\mathrm{d}y$, thermal displacement thickness [m];
ϕ, angle of inclination to vertical; positive for an upward facing plate, negative for a downward facing plate;
η, $\dfrac{y}{x}\left[\dfrac{Gr_x^* Pr\cos\phi}{5}\right]^{1/5}$;
ν, kinematic viscosity [m^2/s].

Subscripts

w, conditions at the wall;
∞, ambient, i.e. conditions far away from wall.

INTRODUCTION

THE STUDY of free convection from an inclined plate has received comparatively little attention compared with the classical study of the vertical plate. However, investigations of laminar free convection from inclined plates [1–5] have produced heat-transfer correlations using plate surface temperature readings, mostly with a heated plate at a uniform temperature. A theoretical analysis based on perturbation theory was also presented in [2] which covered a range of angles from $-60° \leqq \phi \leqq 60°$. All these papers showed that the vertical plate theory was adequate provided the gravitational force parallel to the plate was used.

Transition from laminar to turbulent flow has been studied experimentally in [6, 7] for isothermal inclined plates. More recently it has been shown theoretically [8] that the neutral stability curves are displaced to lower Grashof numbers on going from a downward to an upward facing plate.

Turbulent free convection on a vertical plate has been studied by several investigators [9–11]. For an upward facing inclined plate [3] showed that the heat-transfer coefficient was independent of plate angle whereas [12], using mass-transfer techniques, showed that it was proportional to the angle of inclination. However, no data for turbulent flow are available for the downwards facing case.

This paper presents a study of free convection from a downward facing inclined plate. The results extend the data for a uniform-heat-flux plate in the laminar regime and include a new transition criterion and novel turbulent flow heat-transfer data.

EXPERIMENTAL METHOD

The experiments were carried out in a water tank, 1.83 m long by 1.83 m high by 0.76 m wide. On one of the larger sides of the tank a plate-glass observation window was installed and the adjacent side wall had a narrow perspex window. The heated plate, 1 m long and 0.61 m wide, consisted of an electrically-heated copper–nickel foil backed by silicone rubber and a phenolic sheet. The heating element consisted of five strips of 0.05 mm thick and 12.2 cm wide foil heated by alternating current. The strips were placed adjacent to each other with a spacing of 2 mm between them and connected in series. The variation of thickness of the

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foil produced only $\pm 6\%$ variation in energy dissipation. The centre foil was the test section with the remaining four foils acting as guard heaters. Thirty-three insulated copper–constantan thermocouples were attached to the inner surface of the test section and twelve across the plate to check for any variation due to edge effects. The power supply gave a stepless output of 0–82 V (50 Hz) with a maximum current of 375 A.

The outputs from the thermocouples on the heated plate and also in the bulk fluid were recorded on magnetic tape using a data logging system. Subsequently the signals were processed from analogue to digital form and averaged by a mini-computer. Temperature distributions were investigated using a traversing thermocouple probe. The probe traversed the boundary layer perpendicular to the plate. The thermocouple probe was made from 0.2 mm diameter copper–constantan wires, and the probe could be positioned to within 0.05 mm.

Because of the low frequency fluctuating temperatures encountered in the transition and turbulent regimes, the signal from the temperature probe was amplified and recorded on magnetic tape. The signal was recorded for a period of 100 s and then digitally sampled every 0.1 s with the appropriate statistics being obtained by averaging over 1000 samples.

Experience with a smaller rig [13] had shown that care should be taken to ensure that no unwanted flows were present in the main bulk of the fluid away from the plate. Therefore, bulk flow patterns outside the boundary layer were observed by photographing small neutral density polystyrene particles suspended in the water and illuminated by a light source. Initial flow visualization showed that with the rig running at full power, several large recirculating vortices were produced. To prevent the vortices forming, a baffle and cooler system was used (see Fig. 1).

The plume of water leaving the trailing edge of the plate rose to the surface of the water and extended along the top of the tank producing a vortex. An adjustable baffle was used to prevent the vortex disturbing the main bulk of the fluid. At the top right-hand corner of the tank, a tubular heat exchanger was used to cool the water. The cooled water flowed slowly down a passage at the far end of the tank opposite the heat transfer surface, and returned slowly into the main bulk of the fluid. The constraints of finite size were thus avoided.

As the position of the plate was changed, it was necessary to change the angle of the adjustable baffle to retain the gap between the trailing edge of the heated plate and the edge of the baffle. For angles less than $-45°$ the adjustable baffle was not required, a vortex forming above the plate but not interfering with the flow beneath the plate. At angles less than $-80°$ the plate was moved higher in the tank so that a larger volume of water was beneath the plate and the complete baffle system was removed but the cooler retained.

De-aerated, de-ionized water was used for all experiments to avoid air bubbles forming on the heated plate and to minimise electrical leakage through the water. After the angle of inclination of the plate had been set, the water was thoroughly stirred to ensure a uniform temperature in the tank. The heated plate was then switched on after the disturbances in the water had died away. After the flow had reached a quasi-steady state, the difference between the centre line thermocouples and those near the plate edges was less than 5% for all angles of inclination, indicating that the heat-transfer phenomena were approximately two-dimensional.

LOCAL HEAT-TRANSFER MEASUREMENTS

The temperature distribution along the heated plate surface was investigated for a range of heat flux density and angle of inclination. Figure 2 shows the wall temperature distribution along the vertical plate. As the heat flux density was increased, the point of transition occurred nearer to the leading edge of the plate. In the turbulent region there was a gradual increase in temperature along the plate. The variation of wall temperature distribution against distance along the plate for three angles of inclination is shown in Fig. 3. The three cases have approximately equal heat

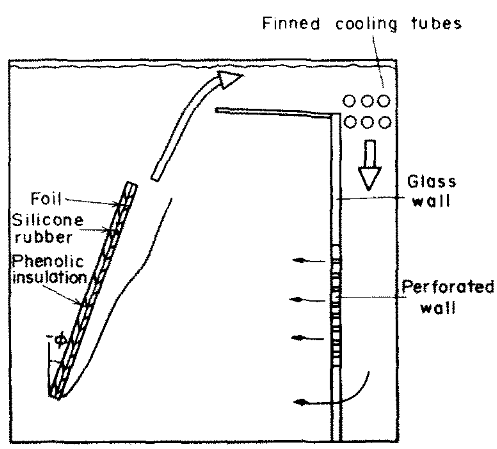

FIG. 1. Configuration of heated plate, cooler and baffle.

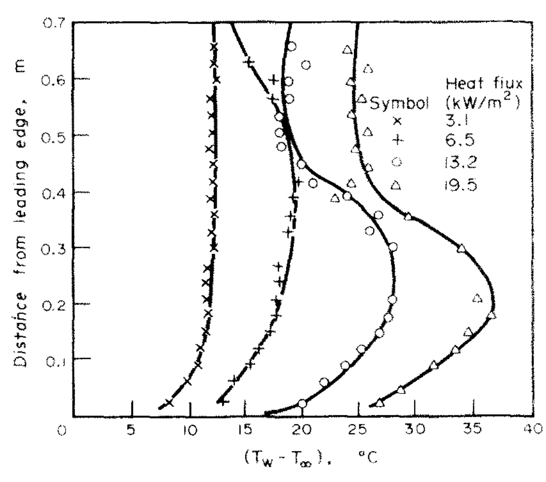

FIG. 2. Wall temperature distributions along the vertical plate.

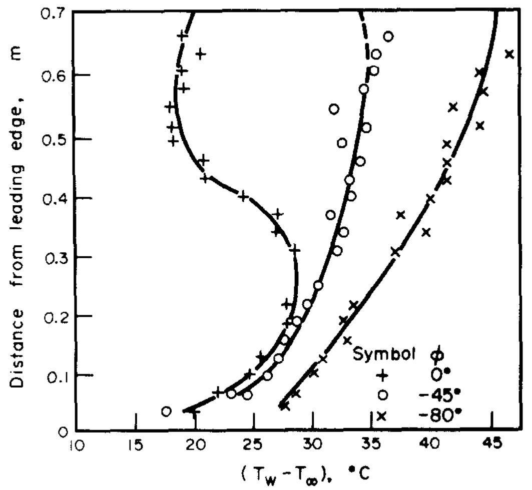

FIG. 3. Wall temperature distributions along the plate for inclinations of 0, −45 and −80°.

fluxes. As the angle of inclination changed from the vertical to −80°, the point of transition moved further up the plate. This is to be expected because the flow becomes more stable, the further the plate is from the vertical.

Laminar flow data

Heat-transfer measurements were obtained for angles of inclination from 0 to −86.5° and heat flux densities from 3 to 20 kW/m^2. The data were correlated and the following equation was obtained:

$$Nu_x = 0.592(Gr_x^* Pr \cos\phi)^{1/5} \qquad (1)$$

with a standard deviation of 7.47%. Correlating the data with equations derived by an approximate theoretical solution [13, 14] resulted in the following two expressions:

$$Nu_x = 0.630Pr^{2/5}(Pr + 0.8 - 0.15Pr\sin\phi)^{-1/5} \times (Gr_x^*\cos\phi)^{1/5} \qquad (2)$$

and

$$Nu_x = 0.619Pr^{2/5}(Pr+0.8)^{-1/5} \times [1 + 0.798 f(Pr, Gr_x^*, \phi)\tan\phi]^{1/5} \times (Gr_x^*\cos\phi)^{1/5} \qquad (3)$$

where

$$f(Pr, Gr_x^*, \phi) = Pr^{-2/5}(0.8 + Pr)^{1/5}(Gr_x^*\cos\phi)^{-1/5}$$

both with standard deviations of 7.46%. The various correlations are compared in Fig. 4 for $Pr = 5$.

The theoretical solutions of [13, 14] predict a coefficient of 0.600 in equation (1) compared with the value of 0.592 presented here and 0.546 presented earlier [13]. The lower value of [13] is probably caused by the varying heat flux along the plate surface which gave a greater scatter and also gave a shift toward the isothermal boundary condition (for which lower values of heat transfer are to be expected). An attempt to correlate the data of [13] into the form of equation (2) resulted in a large coefficient for the sin ϕ term, and when a correlation of the form of equation (3) was attempted, no satisfactory value of the coefficients could be found to cover the range of inclination. This is believed to be caused by stratification in the main bulk of the fluid, which changes the heat-transfer coefficient [15] and [16]. The stratification in the bulk fluid in [13] was ~5°C/m compared with ~1°c/m in the present study. The variation of stratification along the plate as the angle of inclination is changed accounts for some of the difficulty experienced in correlating results over a wide range of inclination. For the smaller rig used in the earlier work [13], the results of Schwind and Vliet [17] suggest that the heat-transfer data were obtained in an "intermediate stratification" stage where the plume of hot water leaving the plate is beginning to penetrate the unstratified region.

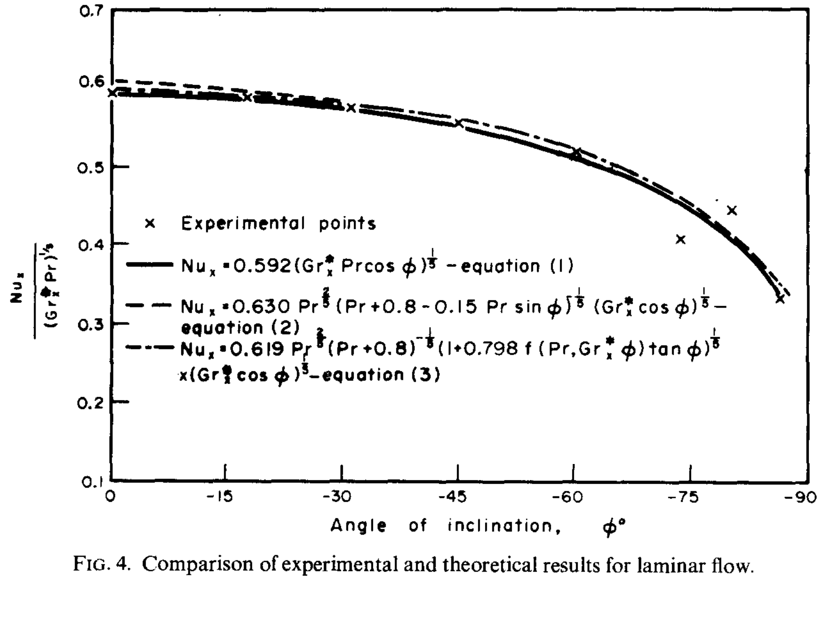

FIG. 4. Comparison of experimental and theoretical results for laminar flow.

Transition data

Theory predicts for laminar flow that $Nu_x \propto (Gr_x^* Pr)^{1/5}$. By plotting the heat-transfer results as a graph of $Nu_x/(Gr_x^* Pr)^{1/5}$ against log $(Gr_x^* Pr)$, as suggested by [12], it is easier to see whether this relationship is obeyed by the data and the point of transition is made much clearer. This criterion was applied to the results for a range of angles of inclination, and Fig. 5 shows the variation of transition Rayleigh number with angle of inclination. Other techniques have been used to establish transition. For example, Tritton [18] used a fibre anemometer to detect local turbulence, and Lock *et al.* [6] used local thermocouple probes to assess transition. However, the transition data of Vliet [3] for uniform-heat flux, which are also shown on Fig. 5, are based on the location of the decrease in wall temperature associated with transition. The agreement between the present study and [3] at the vertical position indicates reasonable compatibility between these different criteria for transition, although comparison with methods based on the detection of local disturbances could produce large discrepancies.

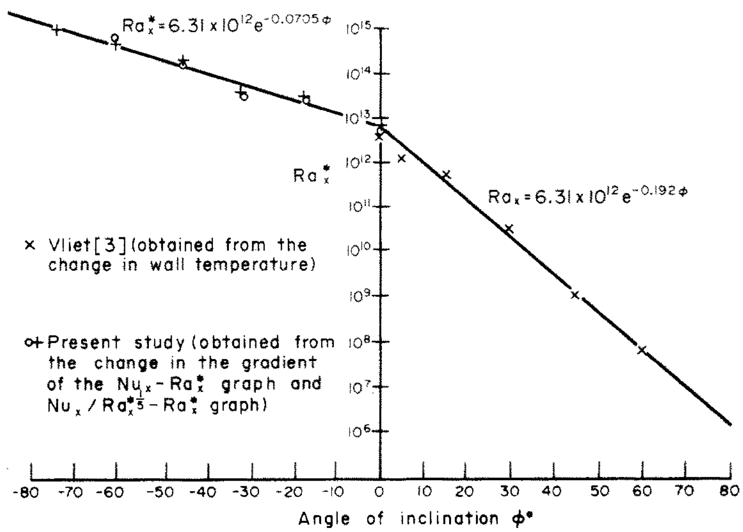

FIG. 5. Transition data for upwards and downwards facing plates.

The following correlation was derived to calculate the transition Rayleigh number for a downward facing plate

$$Ra_x^* = 6.31 \times 10^{12}\, e^{-0.0705\phi} \qquad (-70^\circ < \phi < 0^\circ). \qquad (4)$$

Turbulent flow

Turbulent heat-transfer data were obtained for angles of inclination of 0, -17.5 and -31.67° and covered a range of Rayleigh numbers of 5.3×10^{13} to 9.23×10^{14}. The best fit curve through the data was:

$$Nu_x = 0.889(Gr_x^* Pr \cos\phi)^{0.205} \qquad (5)$$

with a standard deviation of 6.59%. It was found that the gravity component parallel to the plate correlated the data better than the ordinary gravity component (cf. [3]). Figure 6 shows the turbulent heat-transfer data.

There is little agreement on the power of the Rayleigh number amongst the various investigations that have been undertaken. Published data reveal a wide range of values for the index of Gr_x^*, values from 0.22 to 0.29 being quoted [3, 9–12, 19]. The slope of the data for the present investigation is less than the other correlations but the data points lie within the large band of scatter of the correlations.

The difference in turbulent heat-transfer coefficients between an upward and downward-facing inclined plate may be due to the separation of the boundary layer from the plate which occurs for an upward-facing plate but not for a downward-facing one. The separation could cause an increase in heat-transfer coefficient.

Figure 7 gives a comparison of the local heat-transfer correlations for the laminar transition and turbulent regimes. The increase in Nu_x in going from laminar to turbulent flow is displayed.

BOUNDARY-LAYER TEMPERATURE PROFILES

Laminar boundary-layer temperature profiles were obtained over a range of angles from 0 to -88.5° and Rayleigh numbers from 2.2×10^{10} to 1.9×10^{14}. The heat flux and plate surface temperatures were obtained by extrapolating the temperature to the wall, the gradient of the curve giving the heat flux. Agreement within 5% was obtained with the heat flux and plate surface temperature obtained from the plate surface thermocouples.

The temperature data were normalised using $(T - T_\infty)/(T_w - T_\infty)$ and the distance from the plate by $(y/x)[Gr_x^* Pr \cos\phi/5]^{1/5}$. Figure 8 shows a selection of results for the variation of normalised temperature as a function of distance from the wall for angles of

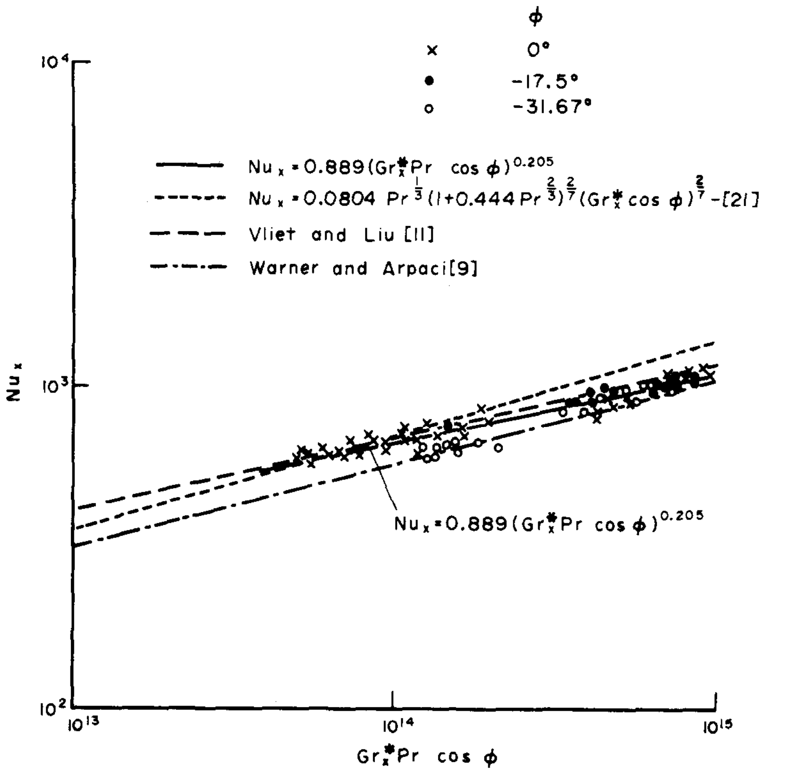

FIG. 6. Turbulent free convection correlations for an inclined flat plate.

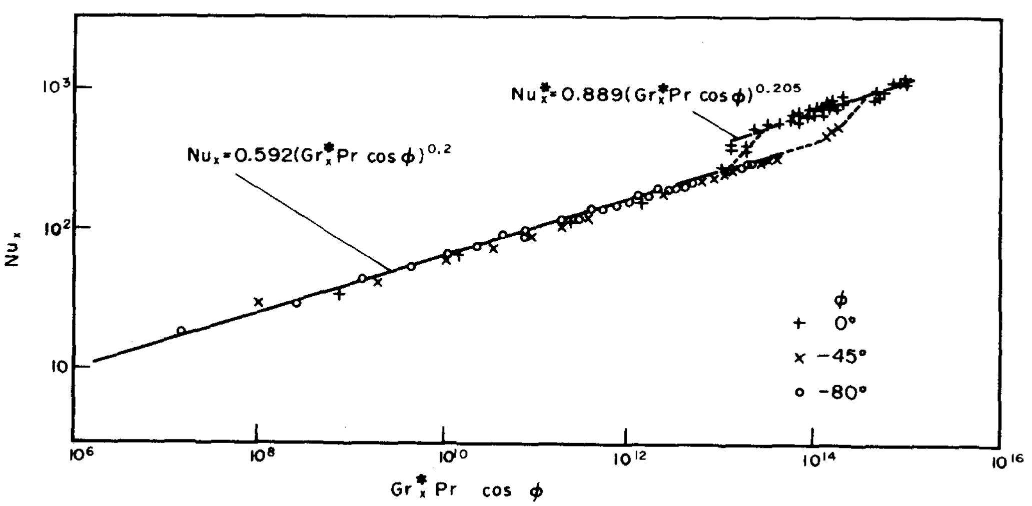

FIG. 7. Laminar and turbulent heat-transfer correlations for free convection from an inclined plate.

inclination of 0, -16.83, -60 and $-85°$ from the vertical. The data covered a range of Prandtl numbers from 4.5 to 6.5. The experimental data were also compared with the vertical plate theory of [20].

For the vertical plate, there was excellent agreement between theory and experiment. As the plate moved towards the horizontal, the experimental points began deviating from the theoretical curve up to $-80°$ from the vertical. At angles between -80 and $-88°$, the results taken at small x were below the curve but at higher x they were above the curve.

An alternative method of presenting the results [9], which employs the thermal displacement thickness, is used in Fig. 9 which shows the thermal boundary-layer profiles for laminar and turbulent cases. There is a steeper profile for turbulent flow. This figure also compares the vertical and inclined plate mean turbulent boundary-layer temperature profiles with the data of Vliet and Liu [11]. Good agreement was found between the two sets of data.

Temperature fluctuations were measured at various points throughout the boundary layer and a typical set

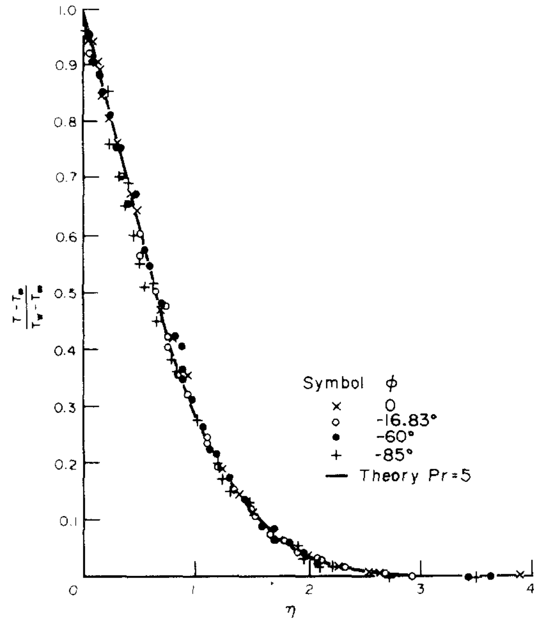

FIG. 8. Laminar boundary-layer temperature profiles.

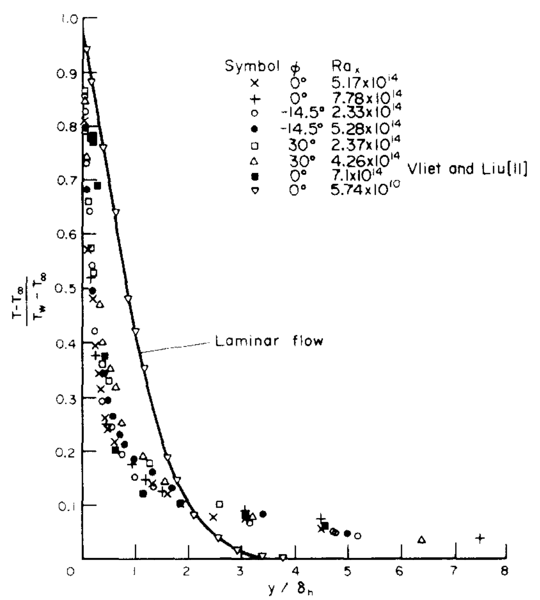

FIG. 9. Mean temperature distributions in the turbulent boundary layer.

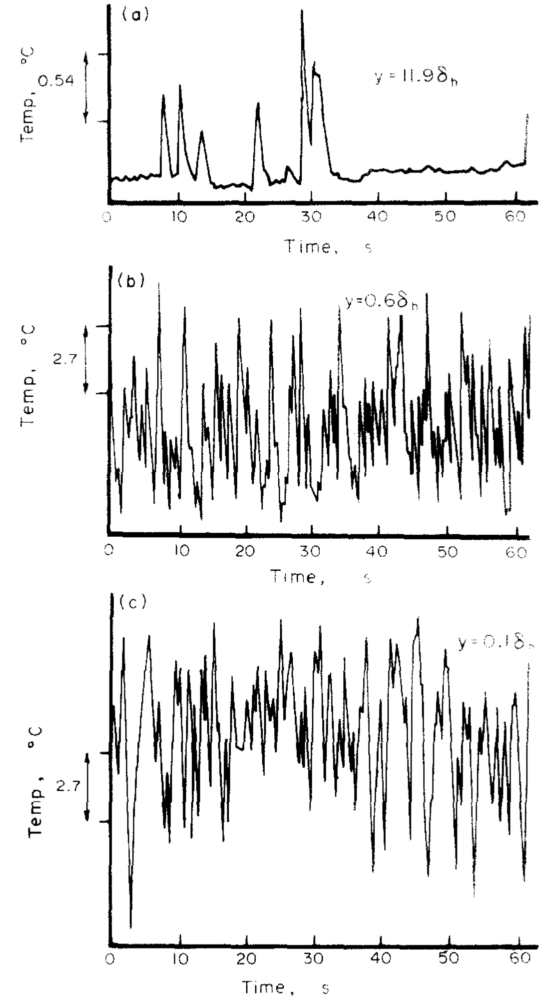

FIG. 10. Temperature fluctuations in the boundary layer.

of traces is shown in Fig. 10 for an angle of inclination of $-14.5°$ and $Ra_x^* = 5.28 \times 10^{14}$. Figure 10a, taken at the outer edge of the boundary layer, shows intermittency typical of that described in [18] as localized bursts of turbulence which spread into surrounding laminar fluid. As one proceeded towards the wall, the asymmetry began to disappear (Fig. 10b) until at $y = 0.1\delta_h$ (Fig. 10c) the turbulence was symmetric. However, the lack of compensation for the thermal inertia of the probe precludes quantitative interpretation of these results.

The RMS values of the turbulent temperature fluctuations as a function of y/δ_h are presented in Fig. 11. The intensity increased as one proceeded towards the wall until it reached a maximum of $y/\delta_h = 0.25$ where it began to decrease. A graph of turbulent velocity fluctuations in the boundary layer from Vliet and Liu [11] showed a maximum at a similar value.

CONCLUSIONS

An experimental study has been made of quasi-two-dimensional free convection heat transfer from a downward-facing inclined flat plate and the following conclusions were obtained.

(i) Local laminar heat-transfer measurements were found to agree well with theory and other workers. To a first approximation, the data may be correlated using the vertical plate theory modified to include the gravitational component parallel to the plate. For angles less than $-70°$ the data began to deviate from this correlation and the following alternative overall correlations are proposed to cover the range 0 to $-86°$.

$$Nu_x = 0.630Pr^{2/5}(Pr+0.8-0.15Pr\sin\phi)^{-1/5} \times (Gr_x^* \cos\phi)^{1/5}$$

and

$$Nu_x = 0.619Pr^{2/5}(Pr+0.8)^{-1/5} \times [1+0.798f(Pr, Gr_x^*, \phi)\tan\phi]^{1/5} \times (Gr_x^*\cos\phi)^{1/5}.$$

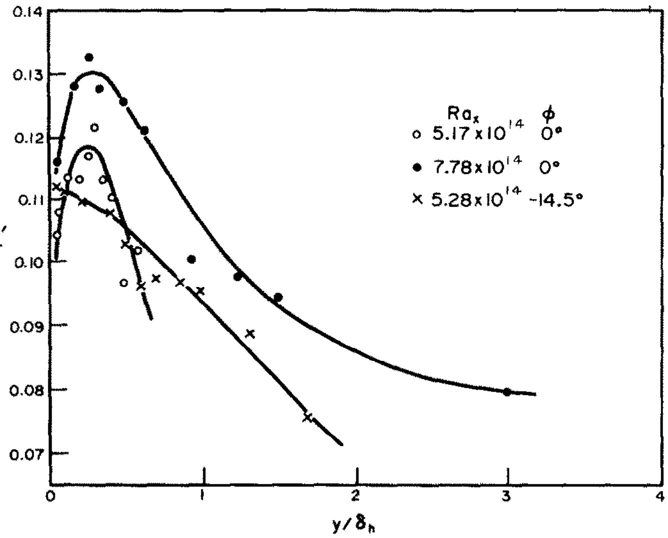

FIG. 11. Root mean square temperature fluctuations.

The coefficients of the $\sin\phi$ and $\tan\phi$ terms in equations (2) and (3) increase with increasing stratification. The correlations above are for a bulk fluid stratification of ~1°C/m. Comparison of these results with earlier measurements [13] is complicated since there is not only a greater stratification in those experiments but also some variation from the uniform-heat-flux condition. The latter is believed to account for the overall increase in Nusselt number from [13] to the present work.

(ii) Transition on a uniform-heat-flux downward facing inclined flat plate was found at a condition determined by the critical Rayleigh number, $Ra_x^* = 6.31 \times 10^{12} \exp(-0.0705\phi)$. The results agreed with previous work at the vertical position.

(iii) In the turbulent regime for free convection heat transfer, the local heat-transfer coefficient may be found from the correlation:

$$Nu_x = 0.889(Gr_x^* Pr \cos\phi)^{0.205}$$

over the range $0 < \phi < -31°$. Although there is considerable variation in the previously published data, this correlation of data falls within the scatter band.

(iv) Boundary-layer temperature profiles were obtained over a wide range of Rayleigh number and angle of inclination. The laminar flow profiles agreed well with the vertical plate theoretical profiles if the gravity component parallel to the plate surface was used.

(v) The turbulent temperature fluctuations in the boundary layers showed a maximum value at about $y/\delta_h = 0.25$, which is similar to the variation in turbulent velocity fluctuations previously reported [11].

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CONVECTION NATURELLE AUTOUR D'UNE PLAQUE PLANE, INCLINEE, A FACE TOURNEE VERS LE BAS

Résumé –On étudie la convection naturelle laminaire ou turbulente dans l'eau, pour une plaque plane, inclinée, à face tournée vers le bas et chauffée à flux uniforme. Un système est employé pour éliminer les effets de volume fini du réservoir d'eau. On présente les résultats du transfert thermique pour les écoulements laminaires et turbulents avec les critères de transition. Les profils moyens de température dans la couche limite et, dans le cas des écoulements turbulents, les fluctuations de température, sont présentés.

FREIE KONVEKTION AN DER UNTERSEITE EINER GENEIGTEN, EBENEN PLATTE

Zusammenfassung –Es wurde die laminare und turbulente freie Konvektion einer geneigten ebenen Platte in Wasser mit gleichmässiger Wärmestromdichte untersucht. Um die Einflüsse des endlichen Volumens des Wasserbehälters zu eliminieren, wurde ein System von Leitblechen und ein Kühler installiert. Es werden Wärmeübergangsgleichungen für die laminare und turbulente Strömung im Zusammenhang mit den Übergangskriterien engegeben. Die mittleren Temperaturprofile in der Grenzschicht und, für den Fall der turbulenten Strömung, die Temperaturschwankungen werden ebenfalls dergestellt.

СВОБОДНАЯ КОНВЕКЦИЯ ОТ НАКЛОННОЙ ПЛОСКОЙ ПЛАСТИНЫ С ОБРАЩЕННОЙ ВНИЗ ПОВЕРХНОСТЬЮ ТЕПЛООБМЕНА

Аннотация — Исследовалась ламинарная и турбулентная свободная конвекция в воде от наклоненной вниз нагретой стороной плоской пластины при однородном тепловом потоке. Система перегородок и охладитель были применены с целью устранения влияния размеров объема резервуара с водой. Представлены соотношения для теплообмена в условиях ламинарного и турбулентного течения, а также найден критерий перехода. Приводятся средние профили температуры в пограничном слое, а в случае турбулентного потока и пульсации температуры.